Hyperedge2vec: Distributed Representations for Hyperedges

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Outline

Motivation

- Group structured data is more abundant than studied
- Examples:
  - Social: MMO Game or Software Teams, research collaborations, communication tools like Skype, etc.
  - Others: NLP (sentences), Biology (protein complexes), e-commerce (item-sets) and Chemistry (reaction species).

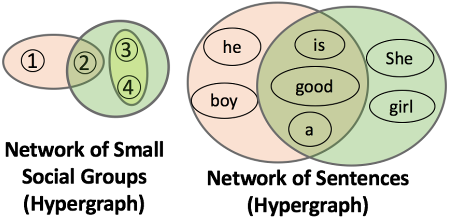

Figure 1: Left is a collaboration network between four individuals and on right is a network resulting from two sentences: "He is a good boy" & "She is a good girl"; and eight word nodes.

Representing Group Structure

- *Hypergraph* is a generalization of graphs
- Naturally captures higher-order relationships between sets of objects
- Hypergraph also has a corresponding *Hasse Diagram*

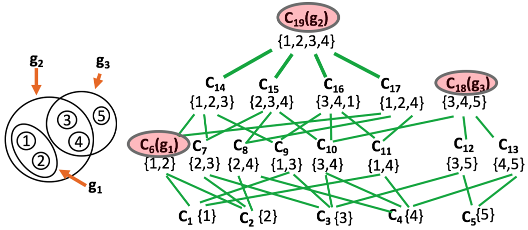

Figure 2: Example of a hypergraph (left) and the hasse diagram (right) corresponding to this hypergraph's simplicial complex.

Problem Statement

- Input:
  - $V = \{v_1, v_2, ..., v_n\}$ represents n elements (actors or words)
  - $G = \{g_1, g_2, ..., g_m\}$, where $g_i \subseteq V$ is a set (group or sentence)
  - Each $g_i \in G$ has occurred $R(g_i)$ times
  - As these sets are overlapping we consider them as a hypergraph $N_g = (V, G)$
  - Incidence matrix $\mathbf{H} \in \{0, 1\}^{|G| \times |V|}$ associated to N_g, with $\mathbf{H}(g_i, v) = 1$ if $v \in g_i$ else 0.
- Goal: Learn the mapping $\mathbf{Z} : G \to \mathbb{R}^d$ from hyperedges to feature representations (i.e., embeddings)

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  - *Preserving set-level info.:* Capture hypergraph topology in a loss-less manner (**Addresses 2 & 3**) [Shashua et al., 2006, Bulò and Pelillo, 2009, Ghoshdastidar and Dukkipati, 2017].

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- But, both completely ignore hypergraph structure (**unaware!**)

Methods

- We propose two methods in order to:
  1. Directly learn hyperedge embeddings
  2. For different cardinality hyperedges simultaneously (i.e. non-uniform hypergraphs)
  3. Capture the hypergraph structure in a principled manner
  4. Retain the hyperedge-level higher-order information

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- **First**, is an **algebraic method** which is based on the novel *dual* (**addresses 1**) tensors (**addresses 4**) of different sizes decomposed simultaneously (**addresses 2**) while regularized by the hypergraph topology (**addresses 3**)
- **Second**, is **neural network** based deep (nonlinear, possibly **addresses 4**) auto-encoder which embeds each hyperedge vector (naturally **addresses 1 & 2**) with the noise generated from hypergraph's *hasse* topology (**addresses 3**)

An Information Theoretic Result from Combinatorics

Proposition

Given a set of random variables $X_1, ...X_c$ $(c \geq 2)$, and $H(.)$ as the information entropy, we have ([Chung et al., 1986, p. 34]):

$$H(X_1, ..., X_c) \leq (\frac{1}{c-1}) \sum_{(i,j) \subseteq 2^{[c]}} H(X_i, X_j)$$

Therefore, the joint probability distribution over c cardinality hyperedges is more informative (lower entropy) than the sum total of information attained from probability distributions over each of the $\binom{c}{2}$ dyadic edges. $\Rightarrow$ Tensors should retain set-level information

Hyperedge2vec using Hypergraph Tensor Decomposition

- Hypergraph $N_g \xrightarrow{\text{extract}} k$-uniform sub-hypergraph or k-graph

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- k-graph $\Rightarrow$ k^{th} order n-dimensional symmetric tensor $\mathcal{A}^k_{\textbf{hyp}} = (a_{p_1,p_2,..,p_k}) \in \mathbb{R}^{[k,n]}$ with $a_{p_1,p_2,..,p_k} = R(g_i)$, where $\{v_{p_1}, v_{p_2}, ..., v_{p_k}\} \in g_i$ and $|g_i| = k, \forall i \in \{1, ..., m\}$.

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- *Symmetry* $\Rightarrow a_{p_1,p_2,..,p_k}$ is invariant under any permutation of its indices $(p_1, p_2, .., p_k)$.

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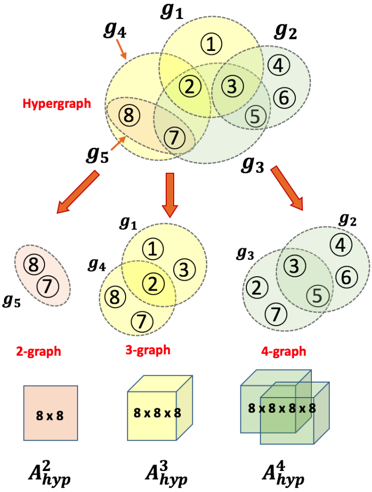
- *Symmetry* $\Rightarrow$ $a_{p_1,p_2,..,p_k}$ is invariant under any permutation of its indices $(p_1, p_2, .., p_k)$.
- Define the lexicographically ordered index set for hyperedges:

$$\mathcal{P}^k = \{\mathbf{p}|\mathbf{p} = (p_1, p_2, .., p_k) \text{ where } \{v_{p_1}, v_{p_2}, ..., v_{p_k}\} \in g_i, \\ \forall g_i \in G \text{ s.t. } |g_i| = k \text{ and } p_1 < p_2 < ... < p_k\},$$

and we have different sets $\{\mathcal{P}^k\}, \forall k \in \{c_{min}, .., c_{max}\}$ (cardinality range). Also, we have $|\mathcal{P}^k| = |\{g_i : |g_i| = k\}|$.

Example Hypergraph Tensor

- $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- $G = \{g_1, g_2, g_3, g_4, g_5\}$
- $\mathcal{A}^2_{\textbf{hyp}}$, $\mathcal{P}^2 = \{(7, 8)\}$
- $\mathcal{A}^3_{\textbf{hyp}}$, $\mathcal{P}^3 = \{(2, 7, 8), (1, 2, 3)\}$
- $\mathcal{A}^4_{\textbf{hyp}}$, $\mathcal{P}^4 = \{(3, 4, 5, 6), (2, 3, 5, 7)\}$



Hyperedge2vec using Hypergraph Tensor Decomposition

- Dual Hypergraph $N_d \xrightarrow{\text{extract}} k$-uniform dual or k-dual

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- Lexicographically ordered index set for *dual* hyperedges (vertices in the *original* hypegraph):

$$\mathcal{Q}^k = \{\mathbf{q}|\mathbf{q} = (q_1, q_2, .., q_k) \text{ where } v_j \in \{g_{q_1}, g_{q_2}, ..., g_{q_k}\}, \\ \forall v_j \in V \text{ s.t. } |d(v_j)| = k \text{ and } q_1 < q_2 < ... < q_k\},$$

and we have different sets $\{\mathcal{Q}^k\} \forall k \in \{d_{min}, .., d_{max}\}$ (vertex degree range in the *original* hypergraph). We have $|\mathcal{Q}^k| = |\{v_i : d(v_i) = k\}|$

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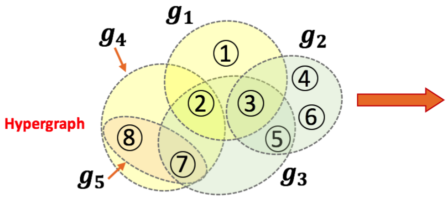

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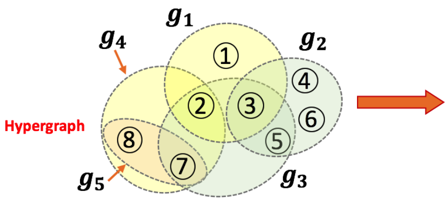

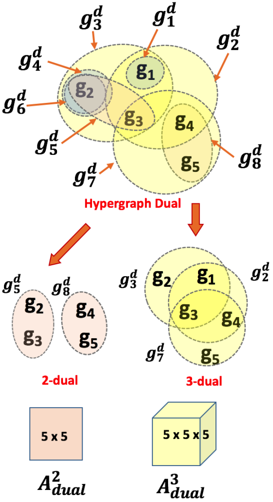

- $V_{dual} = \{g_1, g_2, g_3, g_4, g_5\}$
- $G_{dual} = \{g_1^d, g_2^d, g_3^d, g_4^d, g_5^d, g_6^d, g_7^d, g_8^d\}$
- $\mathcal{A}^2_{\mathbf{dual}}$, $\mathcal{P}^2 = \{(2,3), (4,5)\}$
- $\mathcal{A}^3_{\mathbf{dual}}$, $\mathcal{P}^3 = \{(1,2,3), (1,3,4), (3,4,5)\}$

Hyperedge2vec using Hypergraph Tensor Decomposition

- For the hyperedge embeddings we consider the following optimization formulation:

$$f(\lambda, \mathbf{Z}) = \sum_{k=\alpha_1}^{\alpha_2} D_{\mathrm{KL}}\left(\mathcal{M}^k \middle\| \mathcal{A}^k_{\textbf{dual}}\right)$$

where,

$$\mathcal{M}^k = \sum_{r=1}^{d} \lambda_r (\underbrace{\mathbf{z}_r \otimes \mathbf{z}_r \otimes \ldots \otimes \mathbf{z}_r}_{k \text{ times}}) \equiv \sum_{r=1}^{d} \lambda_r \mathbf{z}_r^{\otimes k}$$

with $\otimes$ is the Kronecker product (generalized outer product), $\mathbf{z}_r \in \mathbb{R}^m$, $\lambda_r \in \mathbb{R}$, $\mathbf{Z} \in \mathbb{R}^{m \times d}$ with $\mathbf{Z}(:, r) = \mathbf{z}_r$.

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Hyperedge2vec using Hypergraph Tensor Decomposition

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Employs topology to (1) address cold start issues (2) gluing embeddings from various k-duals

$$f(\lambda, \mathbf{Z}) = \sum_{k=\alpha_1}^{\alpha_2} D_{\mathrm{KL}}\left(\mathcal{M}^k \middle\| \mathcal{A}^k_{\mathbf{dual}}\right) + \eta \sum_{r=1}^{d} \mathbf{z}_r^{\mathsf{T}} \mathbf{L}_{\mathbf{dual}} \mathbf{z}_r$$

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Example Hypergraph Tensor Decomposition

$$f(\lambda,\mathbf{Z}) = \underbrace{D_{\mathrm{KL}}\left(\sum_{r=1}^{d} \lambda_r \mathbf{z}_r^{\otimes 2} \,\middle\|\, \mathcal{A}^2_{\mathbf{dual}}\right) + D_{\mathrm{KL}}\left(\sum_{r=1}^{d} \lambda_r \mathbf{z}_r^{\otimes 3} \,\middle\|\, \mathcal{A}^3_{\mathbf{dual}}\right)} + \eta \underbrace{\sum_{r=1}^{d} \mathbf{z}_r^{\intercal} \mathbf{L}_{\mathbf{dual}} \mathbf{z}_r}$$

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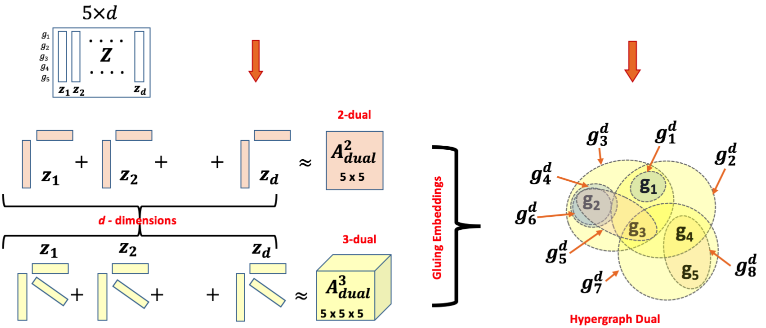
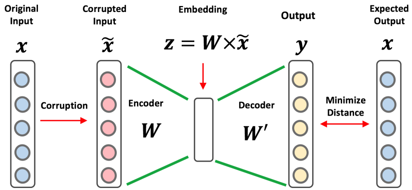


Hyperedge2Vec using Hasse De-noising Auto-encoder



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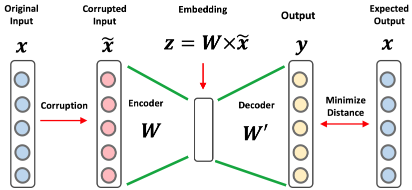

$$\mathbf{W}^*, \mathbf{W}'^* = \underset{\mathbf{W},\mathbf{W}'}{\arg\min} \frac{1}{m} \sum_{i=1}^{m} L(\mathbf{x}_i, \mathbf{y}_i) = \underset{\mathbf{W},\mathbf{W}'}{\arg\min} \frac{1}{m} \sum_{i=1}^{m} L\{\mathbf{x}_i, \sigma(\mathbf{W}'(\sigma(\mathbf{W}\tilde{\mathbf{x}}_i)))\}$$

where $\sigma(x) = 1/(1+e^{-x})$ is sigmoid function, L is the *cross-entropy loss*:

$$L(\mathbf{x}, \mathbf{y}) = \sum_{j=1}^{n} [\mathbf{x}(j) \log \mathbf{y}(j) + (1 - \mathbf{x}(j)) \log(1 - \mathbf{y}(j))]$$

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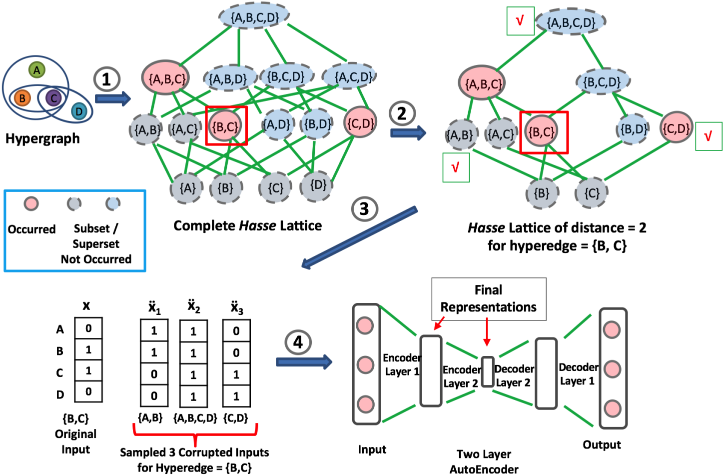

Datasets and Baselines

- Two Datasets:
  - *EverQuest II* (**EQ II**): 5964 hyperedges (teams) among 6536 nodes (players)
  - *Stanford Sentiment Treebank* (**LangNet**): 141,410 hyperedges (phrases) and 21,122 nodes (words)
- Two Proposed Methods:
  - *hypergraph tensor decomposition* (**t2v**)
  - *hypergraph auto-encoder* (**h2v-auto**)
- Six Baselines:
  - *Language Embeddings*: (1) **h2v-DM** (2) **h2v-DBOW**
  - *Spectral Embeddings*: (1) **h2v-inv** (2) **h2v-dual**
  - *Graph Embeddings*: (1) **e2v** (2) **e2v-hyp**
- Except for **h2v-auto** all the baselines and **t2v** can generate both vertex as well as hyperedge embeddings.

Performance of Hypergraph Tensor Decomposition

	Baselines						Hypergraph
	Sentence Embed based		Node2Vec based		Spectral methods		Tensor Decomp.
Embed Combination	**h2v-DM**	**h2v-DBOW**	**h2v-inv**	**h2v-dual**	**e2v**	**e2v-hyp**	**(t2v)**
Node Embed Sum	0.79308	0.79567	0.80418	0.79956	0.81183	0.81405	0.81341
Node Embed Sum + Hyperedge Embed	0.79651	0.80241	0.81362	0.80636	0.8113	0.81652	0.81299
Node Embed Average	0.81584	0.81733	0.82407	0.82281	0.81234	0.81369	0.81303
Node Embed Avg + Hyperedge Embed	0.8182	0.82077	0.83378	0.82896	0.81223	0.81608	0.8127
Only Hyperedge Embed	0.81203	0.81522	0.82189	0.81984	0.81233	0.81608	0.81341

RMSE Scores of **(t2v)** compared to baselines for **EQ II** Team Performance Analysis

	Baselines						Hypergraph
	Sentence Embed based		Node2Vec based		Spectral methods		Tensor Decomp.
Embed Combination	**h2v-DM**	**h2v-DBOW**	**h2v-inv**	**h2v-dual**	**e2v**	**e2v-hyp**	**(t2v)**
Node Embed Sum	0.14081	0.14029	N/A	N/A	0.14633	0.14854	0.14194
Node Embed Sum + Hyperedge Embed	0.14028	0.13883	N/A	N/A	0.14627	0.14845	0.14144
Node Embed Average	0.14245	0.14115	N/A	N/A	0.14665	0.14852	0.14381
Node Embed Avg + Hyperedge Embed	0.14178	0.14007	N/A	N/A	0.14661	0.14845	0.14333
Only Hyperedge Embed	0.14194	0.14147	N/A	N/A	0.14744	0.14844	0.1482

RMSE Scores of **(t2v)** compared to baselines for **LangNet** Sentiment Analysis

Performance of Hypergraph Autoencoder and Run-times

	EQ II			LangNet		
Layer Sizes	**L1:128**	**L1:96/L2:32**	**L1:512/L2:128**	**L1:128**	**L1:96/L2:32**	**L1:512/L2:128**
RMSE	0.81104	0.81512	0.81635	0.14568	0.14529	0.14784
Run Time	52 min	40 min	1 hr 20 min	2 hr 10 min	3 hr 20 min	6 hr

RMSE Scores & Run-times of (**h2v-auto**)

	Baselines						Hypergraph
	Sentence Embed based		Node2Vec based		Spectral methods		Tensor Decomp.
Dataset	**h2v-DM**	**h2v-DBOW**	**h2v-inv**	**h2v-dual**	**e2v**	**e2v-hyp**	**(t2v)**
EQ2	455.84	103.47	90.05	93.41	128.03	12.01	213.37
LangNet	80.61	62.31	211.97*	207.86*	221.46	47.12	483.81

* these are average time taken for learning vertex embeddings only

Average Runtime (seconds) of **(t2v)** compared to baselines across datasets

Choice of Method

	Baselines						Proposed	
	Language Embed		Graph Embed		Spectral Embed		Tensor Embed	Auto-encoder Embed
Property	**h2v-DM**	**h2v-DBOW**	**h2v-inv**	**h2v-dual**	**e2v**	**e2v-hyp**	**t2v**	**h2v-auto**
Interpret-ability	NO	YES	YES	YES	YES	YES	YES	NO
Information Loss	NO	YES	YES	YES	YES	YES	NO	NO
Use Hyp. Topology	NO	NO	YES	YES	NO	YES	YES	YES

Comparing methods

Conclusion

- Propose two hyperedge embedding methods designed specifically for hypergraph data
- Proposed methods embed general hypergraphs, unlike uniform hypergraph which have been the focus in past research
- Introduce the idea of *dual tensors*
- Propose a novel idea of joint decomposition of hypergraph tensors across cardinalities
- Introduce the use of auto-encoder in context of hypergraphs
- Highlight: Leverage the existing structure present in network data as the auxiliary contextual information

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